An Absolute Determination of the Ampere

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From the National Physical Laboratory

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[PLATE 2]

INTRODUCTION

An absolute determination of the ampere has been made with the current weigher erected by AYRTON, MATHER, and SMITH in 1905, modified as described below. A detailed description of this instrument was given at the time of its erection.* It will be sufficient to recall that it is a physical balance with two similar coil systems, one on each side of the central post, fig. 1, Plate 2. Each system consists of two cylindrical coils, of which only the outer one, which has two similar windings separated by a narrow gap at the central diametral plane, can be seen in fig. 1. The small coil is suspended from the end knife-edge and hangs inside the large coil. In the correct position the axes of the two coils are vertical and coincident, and their mean diametral planes coincide.

The current to be measured is maintained in opposite directions in the two parts of the large coil, whereby the small coil is urged up or down, depending on the direction of its own current. The couple is doubled by using the other coil system in the same manner, and the total force is measured by weights in the usual way.

MODIFICATIONS TO THE INSTRUMENT

In the course of the last ten years the balance was thoroughly overhauled and new coils were constructed and erected.

Each marble former is provided with a double-threaded screw groove of 2 mm. pitch, so that each coil consists of two adjacent helical windings which are normally connected in series, but which can also be disconnected for testing the insulation resistance of the marble.

The diameters of the large and small coils, measured between the central filaments of the wires, are nominally 32 cm. and 20 cm. respectively. The axial length of the helical windings of each portion of each large coil is 11 cm. and the distance between the end of one portion and the beginning of the other is 3 cm. The helices of the small coils are 15·2 cm. long.

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* ' Phil. Trans.,' A, vol. 207, p. 463 (1908).

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The wire, of gilt silver 0·56 mm. in diameter, was wound under tension in a manner identical with that employed for the old coils, but the method of securing the ends of the windings is different. A conical bronze pin P, fig. 2, is turned to a size such that, when it is inserted in the radial hole H, and forced in slightly, the wire W which it grips is at the correct radial distance. The winding is then begun. At the other end of the generator, there is a second pin, with its clamp loose. When the winding is complete, the tension on the wire is maintained, and the second pin is forced down to the same level as the first ; this brings its clamp over the end wire, which is gripped by tightening the screw S, and the ends of the wires can then be cut off. Connexions to the terminals are made by soldered joints at projections such as J, J being inside for the small cylinders but outside for the large cylinders.

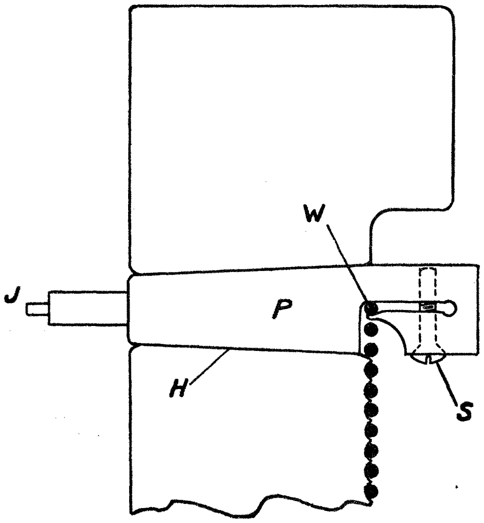

FIG. 2—Pin and clamp for gripping the beginnings and ends of windings.

This construction is very good mechanically, and keeps the wire taut, but the pins, which are about 5 mm. in diameter, prevent the diameter of the coil being measured right at the beginning or the end of the helix.

The diameter of the wire of each coil was measured at a large number of points during the process of winding. It is necessary to know this diameter accurately in order to derive from measurements of the overall diameter of the coils the diameter between the central filaments of the wire, for it is this diameter which enters into the calculations of the electromagnetic force.

For certain measurements with the balance, it is necessary to know accurately the moment of inertia of the moving parts about the fulcrum. To enable this quantity to be determined with accuracy, a fitting capable of carrying a weight at a known distance from the fulcrum was added to the beam. This fitting, which can be seen in fig. 3, Plate 2, is a hollow bronze tube, mounted vertically, and provided at each end with a seating for a bronze ball, which constitutes the weight.

Fig. 3 shows several details of the balance, among them the mirror M by which the image of a distant illuminated wire is thrown on to a scale for reading the position of the beam, the agate balls A which serve to steady the beam without lifting it off the central plane, the flexible silver wire connexions S between the fixed and the moving parts of the circuit, and the amber-mounted commutators C fixed to the spiders from which the small coils are hung. These commutators can be used for disconnecting the adjacent windings of the suspended cylinder, or for reversing the current in one of them.

Measurement of the Linear Dimensions of the Coils.

The mean diameter and the mean axial position of each turn were deduced from measurements made at several points round the circumference.

The cylinder was mounted on a mandrel, and its ends were gripped by cast-iron cheeks fitting the mandrel. The measuring machine used for the diametral measurements was set so that the line perpendicular to the two parallel measuring faces and intersecting the axis of the coil was at right angles to the axis. Thus each measurement was made from a point on one helix to a point on the other helix, and hence the distance measured is actually the sum of the radii of the two helices. The pressure exerted on the wires by the measuring faces is about 4 oz., which does not cause appreciable deformation.

The axial measurements were made with the help of a travelling microscope mounted on an arm carried by a screw calibrated before and after each set of measurements along a generator. The measurements were in the first place referred to the end wire. Subsequently, the distance of that wire from one end face of the marble cylinder was measured, and the mean axial position of each turn expressed as the departure from its nominal position. It was realized in the course of the mutual inductance calculations that, so far as the current balance problem is concerned, it is not necessary to refer axial measurements to the zero faces of the cylinders, because the position of the large cylinder is adjusted so as to make the total force on the small coil a maximum. Reference to a diametral plane is essential, however, when the relative positions of the coils are determined by linear measurements.

The individual diametral measurements were made to the nearest 0·1 micron, the axial measurements to the nearest micron, but the accuracy of the mean dimensions obtained from the measurements is probably much higher that these figures suggest. A comparison of the mean of all the results of the linear measurements with the mean of only half the results indicates that if only half the number of measurements had been taken, the measurement of current would not have been affected by more than two parts in a million. Moreover, the experimental work on the difference between the forces exerted by the two systems of coils shows that the irregularities of the windings have been adequately estimated and allowed for in the calculation.

The end gauges used for the diametral measurements and the line standards used for the axial measurements can be referred to a single standard with an accuracy superior to 0·1 micron, and, in the current balance problem, the unit of length to which they are referred is immaterial.

From these considerations it is deduced that the error in the determination of current, arising from want of exact knowledge of the linear dimensions of the coils, does not exceed 1 part in 100,000.

The thermal coefficients of linear expansion were measured for each cylinder in the axial direction and in directions perpendicular to the axis. If the coefficient were uniform the constant of the instrument would not depend on temperature, for the force between two current-carrying circuits is not altered when, the configuration being preserved, only the size or scale is changed ; actually the values found for the coefficient in the various directions range from $2 \cdot 9 \times 10^{-6}$ to $7 \cdot 4 \times 10^{-6}$ per 1° C., and a correction, which is very small, must be applied ; it depends on the temperature at which the balance is used.

When a cylinder is heated by the current in its windings, there is less cooling surface in the middle portion than at the ends, and thus there is a possibility that the diameter will increase more at the centre than near the ends, in other words, that the cylinder will assume a barrel shape. It was therefore thought advisable to take a few linear measurements under conditions as nearly as possible the same as those obtaining during a measurement of current. The results indicate that the expansions due to the heating effect of the current are small, and, so far as it is possible to measure them, uniform for a given direction. In practice it is therefore sufficient to apply the corrections calculated from the thermal coefficients of expansion for the mean temperature during the experiment.

Calculation of Electromagnetic Forces

In the first place the axial component of the attraction exerted by HG on OP, fig. 4, is calculated for the nominal dimensions. The forces for each individual pair of coils are estimated afterwards by the application of small corrections.

The adjacent helices of any coil start in the same diametral plane, at opposite ends of a diameter, and end on the generators through the starting points. The number of turns on each helix of any one coil is therefore the same, and is integral.

At the beginning and at the end of a helix the wire is connected to a radial pin, and to one of these pins is soldered a straight conductor parallel to the axis ; this conductor would meet the other radial pin if the latter were not cut just short of it. From that point onwards the leads are twisted together, so that the currents in them exert no resultant force on neighbouring current-carrying circuits.

It is sufficient in the first instance to calculate the force between one such complete winding of HG and one of OP, for a difference of azimuth γ between the beginnings of the helical portions of the two windings. The force due to the adjacent windings can then be found by changing γ into $\gamma + \pi$. Provided integrations are carried round closed circuits, the axial component of the attraction between elements ds and ds′ of the windings can be supposed made up of elements, such as

$$f_z = -\{m'(n\mu - m\nu) + l'(n\lambda - l\nu)\}\frac{ds\,ds'}{r^2}, \qquad \ldots\ldots \quad (1)$$

where l, m, n, l', m', n' are the direction cosines of the elements ds, ds', r is the length of the line joining the elements, and λ, μ, ν are its direction cosines. The direction cosines and the length r are easily obtained from the equations of the various parts of the two coils, but the result of the double integration cannot be expressed as an exact formula in terms of known functions and of the azimuthal difference γ. The forms of the integrands indicate, however, the possibility of expressing the integral I_γ in a Fourier series

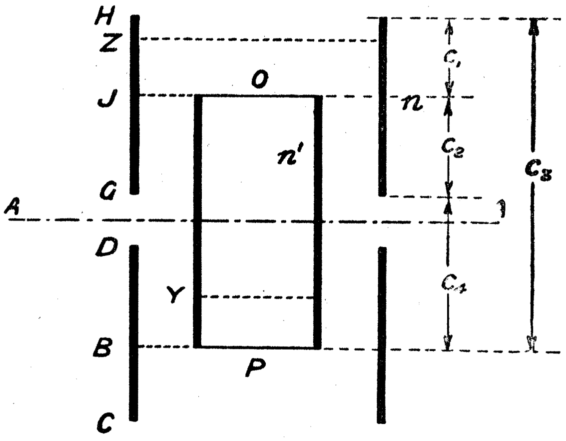

FIG. 4—Arrangement of coils in the current balance of the National Physical Laboratory.

$$I_\gamma = I_{\text{mean}} + \sum_1^\infty (A_n \cos n\gamma + B_n \sin n\gamma), \qquad \ldots\ldots\ldots \quad (2)$$

in which I_{mean} is given by

$$I_{\text{mean}} = \frac{1}{2\pi}\int_0^{2\pi} I_\gamma\, d\gamma. \qquad \ldots\ldots\ldots\ldots \quad (3)$$

The calculation of the coefficients A_n and B_n would be laborious, but the triple integral I_{mean} can be evaluated easily. Calling a, a' the radii of the two coils, n, n' the turns per unit length of each helix, taking c_1, c_2, c_3, c_4 as marked in fig. 4, and writing

$$\nu = -\frac{4aa'}{(a+a')^2},$$

$$k^2 = \frac{4aa'}{\zeta^2 + (a+a')^2},$$

we find

$$I_{\text{mean}} = \phi(c_1) + \phi(c_2) + \phi(c_4) - \phi(c_3), \qquad \ldots\ldots\ldots \quad (4)$$

where

$$\phi\,(\zeta) = 2\pi n n'\zeta \left\{\sqrt{\zeta^2 + (a + a')^2}\,(\mathrm{K} - \mathrm{E}) + \frac{(a - a')^2}{\sqrt{\zeta^2 + (a + a')^2}}\,(\mathrm{K} - \Pi)\right\}, \quad (5)$$

in which K and E are the complete elliptic integrals of the first and second kind, of modulus k, and Π is the integral of the third kind of modulus k and parameter ν.

This is the formula given by Jones* for the force between a helix and a current sheet.

It will be observed that the radial conductors, and those parallel to the axis, do not affect $\mathrm{I}_{\mathrm{mean}}$, but they influence the coefficients A_n and B_n.

The force on one helical coil of OP due to the two helical coils of HG is

$$\mathrm{I}_\gamma + \mathrm{I}_{\pi+\gamma} = 2\mathrm{I}_{\mathrm{mean}} + 2\sum_{1}^{\infty}(\mathrm{A}_{2n}\cos 2n\gamma + \mathrm{B}_{2n}\sin 2n\gamma).$$

To take account also of the second helical coil of OP, this expression must be doubled. Then, writing $\mathrm{F}_{\mathrm{mean}}$ instead of $4\mathrm{I}_{\mathrm{mean}}$, and F_γ for the axial component of the attraction between the complete coils HG and OP, we have

$$\mathrm{F}_\gamma = \mathrm{F}_{\mathrm{mean}} + 4\sum_{1}^{\infty}(\mathrm{A}_{2n}\cos 2n\gamma + \mathrm{B}_{2n}\sin 2n\gamma). \quad \ldots\ldots \quad (6)$$

For the nominal dimensions of the coils, the value of $\mathrm{F}_{\mathrm{mean}}$, calculated from the above formulae, is 945·27217 dynes for a current of 1 ampere.

Since the balance has two similar systems of coils, one at each end of the beam, the effect of change of azimuth can be observed by measuring the difference $\Delta\gamma$ of the forces exerted by the two systems, for different azimuths of one of the large cylinders. From the nature of the problem it is to be expected that the first two periodic terms of the series are more important than the subsequent terms ; retaining these two terms only as a first approximation, we find for the difference between the forces for the two systems

$$\Delta_\gamma = \Delta_0 + \Delta_2\cos 2\gamma + \Delta_2'\sin 2\gamma, \quad \ldots\ldots\ldots \quad (7)$$

in which γ is the azimuthal position of the large cylinder under observation and Δ_2 is a constant difference due to any slight inequality between the two systems of coils.

By measuring the difference of the forces for azimuths 0°, 45°, and 90°, we have

$$\Delta\,(0^\circ) = \Delta_0 + \Delta_2$$
$$\Delta\,(45^\circ) = \Delta_0 + \Delta_2'$$
$$\Delta\,(90^\circ) = \Delta_0 - \Delta_2,$$

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* 'Proc. Roy. Soc.,' vol. 63, p. 204 (1898).

from which Δ_2 and Δ_2' are easily found. The angle $\frac{1}{2}$ arc tan (Δ_2/Δ_2') can then be calculated. When the large cylinder is set at that angle, the force is the same as the mean force F_{mean}.

Measurements on the lines indicated above show that, for a current of 1 ampere, the force for any azimuth γ, is

$$F = F_{\text{mean}} - 0{\cdot}013 \sin 2\,(\gamma - 34^\circ).$$

Thus it appears that with an azimuth of 34° the azimuthal correction becomes zero, and that the force is equal to F_{mean}.

After having calculated F_{mean} for the nominal dimensions of the coils, it is necessary to apply corrections for the departure of each turn of each coil from its nominal diameter and nominal axial position. The turns are helical, but the pitch of the helix is small; in the present case it is only 2 mm. Hence, for the purpose of estimating the corrections due to the small departures of any turn from the nominal diameter and the nominal mean axial position, it is permissible to replace the turn by a coaxial circle in the mean axial position of the turn.

Consider first the diametral corrections. The measurements of the linear dimensions determine the departure from nominal of the mean diameter of each turn of each coil. Let Z, fig. 4, be a turn of the winding HG. Denoting by $M_{HG.O}$ the mutual inductance between the coil HG and a circle of radius a' at O, we have for the attraction F between the coil HG and the coil OP, with unit current in each coil,

$$\begin{aligned} F &= n'\,(M_{HG.O} - M_{HG.P}) \\ &= n' \sum_{HG} (M_{Z.O} - M_{Z.P}) \end{aligned}$$

where $M_{Z.O}$ is the mutual inductance between circles of radii a and a' at Z and O respectively, and the summation extends to every turn of the coil HG.

Departures δa from the nominal radius of the large coil therefore change the force by an amount

$$\delta F = n' \left(\sum_{HG} \frac{\partial M_{Z.O}}{\partial a}\,\delta a - \sum_{HG} \frac{\partial M_{Z.P}}{\partial a}\,\delta a \right) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (8)$$

In the same way, considering a turn of the small coil situated at Y, fig. 4, we have for the attraction between the coil HG and the coil OP,

$$\begin{aligned} F &= n\,(M_{OP.G} - M_{OP.H}) \\ &= n \sum_{OP} (M_{Y.G} - M_{Y.H}) \end{aligned}$$

the summation extending to every turn of the coil OP.

Departures $\delta a'$ from the nominal radius of the large coil therefore change the force by an amount

$$\delta F = n \left(\sum_{OP} \frac{\partial M_{Y.G}}{\partial a'}\,\delta a' - \sum_{OP} \frac{\partial M_{Y.H}}{\partial a'}\,\delta a' \right) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (9)$$

With the notation used previously we have

$$\frac{\partial \mathrm{M}}{\partial a} = 4\pi\sqrt{aa'}\,\frac{k}{2a'}\left(\mathrm{K} - \frac{a+a'}{2a}\,\mathrm{E} - \frac{a-a'}{2ak'^2}\,\mathrm{E}\right), \quad \ldots\ldots \quad (10)$$

and

$$\frac{\partial \mathrm{M}}{\partial a'} = 4\pi\sqrt{aa'}\,\frac{k}{2a}\left(\mathrm{K} - \frac{a+a'}{2a'}\,\mathrm{E} + \frac{a-a'}{2a'k'^2}\,\mathrm{E}\right). \quad \ldots\ldots \quad (11)$$

Formula 10 was used to construct a curve of $\partial\mathrm{M}/\partial a$ for values of z between 0 and 22 cm. From the curve and from the results of the diametral measurements of the large coils $(\partial\mathrm{M}/\partial a)\delta a$ can be found for each turn of each winding of the large coil, and the correction $\delta\mathrm{F}$ obtained by summation, according to formula 8. Proceeding in a similar manner for $\partial\mathrm{M}/\partial a'$, the correction required by the departures of the turns of the small coils from their nominal diameters is obtained from formulae 11 and 9.

Coming now to the pitch corrections, the axial measurements determine the departure of each turn from its mean nominal axial position. For each cylinder they are referred to planes passing through the intersection of the first turn with a particular generator. These planes will be called the " zero " planes. In the calculation of the forces it will be assumed in the first instance that the zero planes of the large and small cylinders are exactly 4·9 cm. apart.

With a reasoning similar to that employed in the preceding section, we find for the correction to be applied for a winding, say HG, of the large cylinder

$$\delta\mathrm{F} = n'\left(\sum_{\mathrm{HG}} \frac{\partial \mathrm{M}_{\mathrm{Z.O}}}{\partial z}\,\delta z - \sum_{\mathrm{HG}} \frac{\partial \mathrm{M}_{\mathrm{Z.P}}}{\partial z}\,\delta z\right). \quad \ldots\ldots\ldots \quad (12)$$

where z is the axial distance between the circles at z and O, and δz is the increment of z due to the departure from nominal of the axial position of the turn at Z. Similarly for the imperfections of the small coil and a perfect winding HG, we find

$$\delta\mathrm{F} = n\left(\sum_{\mathrm{OP}} \frac{\partial \mathrm{M}_{\mathrm{Y.G}}}{\partial z}\,\partial z' - \sum_{\mathrm{OP}} \frac{\partial \mathrm{M}_{\mathrm{Y.H}}}{\partial z}\,\partial z'\right), \quad \ldots\ldots\ldots \quad (13)$$

in which ∂z is the increment of z due to the departure from nominal of the axial position of the turn at Y.

A curve of $\partial\mathrm{M}/\partial z$ for coaxial circles of radii a and a', a distance z apart, is constructed by means of the formula

$$\frac{\partial \mathrm{M}}{\partial z} = -\frac{\pi z k}{\sqrt{aa'}}\left\{2\,(\mathrm{E}-\mathrm{K}) + \frac{k^2}{k'^2}\,\mathrm{E}\right\}. \quad \ldots\ldots\ldots \quad (14)$$

or from published tables, and the corrections are computed from the results of the axial measurements by application of formulae 12 and 13.

It is possible to obtain an experimental check of the accuracy of the pitch and diametral corrections, for, after the cylinders have been adjusted in positions giving maximum force, a known current can be maintained in a direction such that the forces of the two systems exert couples in opposite directions. The small resultant force due to the difference between the two coil systems can be estimated by observing the rest point and determining the sensitivity of the balance. This was done on many occasions, and the difference of force for a current of 1 ampere was found to be 0·317 mg.-wt. The calculated difference is 0·314 mg.-wt. The agreement is excellent, especially in view of the difficulty of measuring a force of only 0·3 mg.-wt. to an accuracy of 3 or 4% on a balance in which the moving parts weigh over 14 kilogrammes.

Let the value of F_{mean} corrected for diameter and pitch be F_c for the force due to the upper portion of the large coil and F_c' for the force due to the lower portion.

The forces F_c and F_c' are calculated for a particular value, in this case 4·9 cm., of the distance between the planes to which the axial measurements of the coils are referred ; but in practice, no attempt is made to set the coils so that the reference planes are exactly this distance apart, for it is much simpler to adjust the relative axial positions of the coils by an electrical method, so as to make the sum of the forces exerted on the small coil by the upper and lower portions of the large coil a maximum. Let the values of the forces in the " maximum " position be F_M and F_M', and let the height of some reference mark of the large cylinder for that position be ξ_M. It is required to find $F_M + F_M'$ when F_c and F_c' are known. Denoting by F_u and F_l the attractions for any other position ξ, we have

$$F_u = F_M + (\xi - \xi_M) \frac{\partial F}{\partial z} + \tfrac{1}{2} (\xi - \xi_M)^2 \frac{\partial^2 F}{\partial z^2} + \ldots \quad . \; . \; . \; . \quad (15)$$

$$F_l = F_M' - (\xi - \xi_M) \frac{\partial F}{\partial z} + \tfrac{1}{2} (\xi - \xi_M)^2 \frac{\partial^2 F}{\partial z^2} - \ldots \quad . \; . \; . \; . \quad (16)$$

the first differential coefficients in the two expressions being equal, since $F_M + F_M'$ is the maximum value of the total force. Subtraction gives

$$F_u - F_l = F_M - F_M' + 2 (\xi - \xi_M) \frac{\partial F}{\partial z} . \quad . \; . \; . \; . \; . \; . \quad (17)$$

It is easy to find by experiment the position ξ_D of the large cylinder for which F_u is equal to F_l. For this position the above formula becomes

$$0 = F_M - F_M' + 2 (\xi_D - \xi_M) \frac{\partial F}{\partial z} . \quad . \; . \; . \; . \; . \; . \; . \quad (18)$$

Moreover, denoting by ξ_c the position of the large cylinder for which the distance between the zero plane of the large cylinder and the zero plane of the small cylinder is exactly 4·9 cm., distance on which the calculation of the forces F_c and F_c' was based, we have by substitution in (17)

$$F_c - F_c' = F_M - F_M' + 2\,(\xi_c - \xi_M)\,\frac{\partial F}{\partial z}. \quad \dots\dots \quad (19)$$

Subtracting (18) from this, we get

$$F_c - F_c' = 2\,(\xi_c - \xi_D)\,\frac{\partial F}{\partial z}. \quad \dots\dots\dots \quad (20)$$

This equation can be used to find ξ_c, since F_c and F_c' are obtained by calculation, ξ_D by experiment, and $\partial F/\partial z$ either by calculation or by experiment.

Putting ξ equal to ξ_c in 15 and 16, and adding, we obtain

$$F_c + F_c' = F_M + F_M' + (\xi_c - \xi_M)^2\,\frac{\partial^2 F}{\partial z^2},$$

and substituting for ξ_c from (20),

$$F_M + F_M' = F_c + F_c' - \left(\xi_D - \xi_M + \frac{F_c - F_c'}{2\partial F/\partial z}\right)^2 \frac{\partial^2 F}{\partial z^2}. \quad \dots\dots \quad (21)$$

The differential coefficients of F can be calculated, but they can also be determined experimentally without difficulty. In fact, the measurements necessary to determine ξ_M also yield $\partial^2 F/\partial z^2$, and the measurements by which ξ_D is determined also yield $\partial F/\partial z$.

In practice, with coils so nearly alike as those of the current balance, ξ_D is so nearly equal to ξ_M, and F_c so nearly equal to F_c', that the difference between $F_M + F_M'$ and $F_c + F_c'$ is hardly detectable. The possibility of determining it experimentally is nevertheless interesting.

The computed value of $F_M + F_M'$ at 18° C. for a current of 1 ampere is 1891·2903 dynes for the right-hand system of coils, and 1890·9817 dynes for the left-hand system. The working current is in the neighbourhood of 1·018 ampere, and thus the total balancing load required in the normal use of the balance is about 4 gm.-wt.

Adjustment of the Positions of the Cylinders

The suspended cylinders are first adjusted to be at approximately the same height, and all four cylinders are levelled. The adjustment for co-axiality can then be proceeded with. Current is maintained in the same direction in the lower windings of each large cylinder and in each small coil and in the direction opposite to this in the upper windings of the large cylinders. The small cylinders are then urged downwards by approximately equal forces. One of the systems being left untouched, the large cylinder of the other system is moved horizontally in a direction y perpendicular to the beam, and the rest point is observed after each change of position.

The relation between change of force and position is thereby obtained, and is found to be parabolic, as in fig. 5. This is in agreement with theory, which gives for the change

$$\delta\,(F_u + F_l) = -\tfrac{1}{2}\,\frac{\partial^2 F}{\partial z^2}\,(\delta y)^2, \qquad (22)$$

where F_u and F_l denote the forces exerted by the upper and lower portions of the large coil.

Adjustment along x, the direction of the beam, is slightly complicated by the secondary forces. In the normal use of the balance each winding of one large cylinder, like HG, fig. 6, besides exerting a force F on the small coil coaxial with it, exerts a force, although a much smaller one, say G, on the small coil hanging from the other end of the balance beam. In the position for maximum, the forces exerted by the two windings of the large cylinder are 2F and 2G respectively. When the direction of the current is such that the direct forces of the two systems produce couples in the same sense, the secondary forces also act cumulatively. The resultant couple is proportional to 4F ± 4G, the change of sign being brought about by reversing the current in all the windings of one system. The sum of the two couples, measured by appropriate weights, is therefore proportional to 8F, and this observed value of 8F, divided by the value of 8F calculated for 1 ampere, gives the square of the current. A knowledge of the secondary force is therefore not essential. The values of G and $\partial G/\partial x$ can be calculated by formulae due to Dr. G. F. C. Searle, F.R.S. (private communication), and agree well with experimental results.

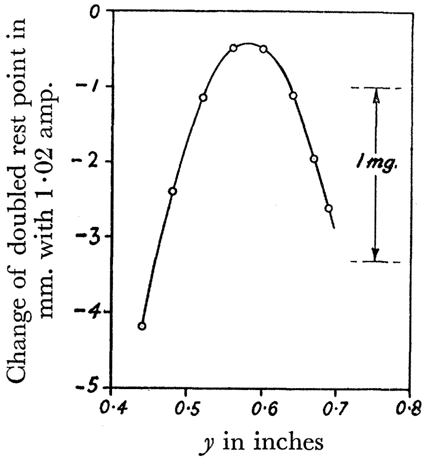

Fig. 5.—Adjustment of the large right-hand cylinder in the direction y perpendicular to the beam.

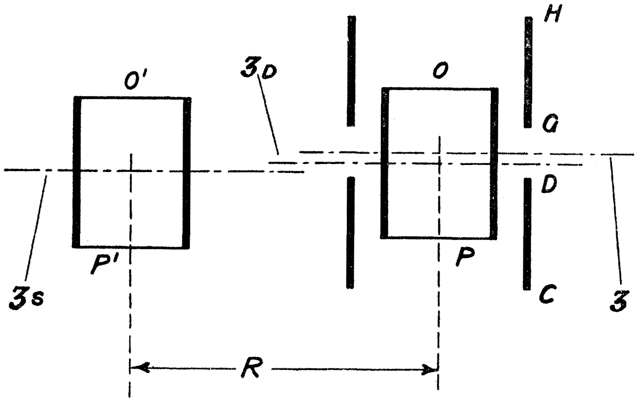

Fig. 6.—Secondary effect.

In the adjustment of the large cylinder in the direction parallel to the beam, a displacement δx of the large cylinder from the coaxial position causes not only a change $\frac{1}{2}\,\frac{\partial^2 F}{\partial z^2}\,(\delta x)^2$ in the force 2F it exerts on the small coaxial coil, but also a change $2\,\frac{\partial G}{\partial x}\,(\delta x)$ in the secondary force 2G exerted on the other suspended coil.

This secondary force is allowed for by taking another set of observations after reversing the current in all the coils of the system under consideration; this does not reverse the direct force, but reverses the secondary force. If the change of force is denoted by Δ, the equations of the curves obtained are

$$\Delta = -\tfrac{1}{2}\,\frac{\partial^2 F}{\partial z^2}\,(\delta x)^2 \mp 2\,\frac{\partial G}{\partial x}\,(\delta x). \qquad \ldots\ldots\ldots \quad (23)$$

The calculated values of $\partial^2 F/\partial z^2$ and $\partial G/\partial x$ for 1 ampere are $-\,25\cdot05$ dynes/cm./cm. and $-\,0\cdot99$ dyne/cm. The curves should therefore be two equal parabolae, with their axes parallel and 0·158 cm. apart. The experimental value deduced from the curves of fig. 7 is 0·062 inch (0·158 cm.). The correct position of the cylinder corresponds to the mean of the abscissae of the axes of the two curves.

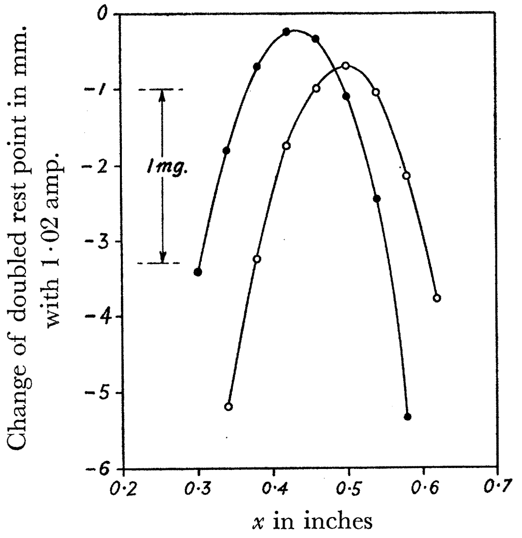

FIG. 7—Adjustment of the large right-hand cylinder in the direction x parallel to the beam.

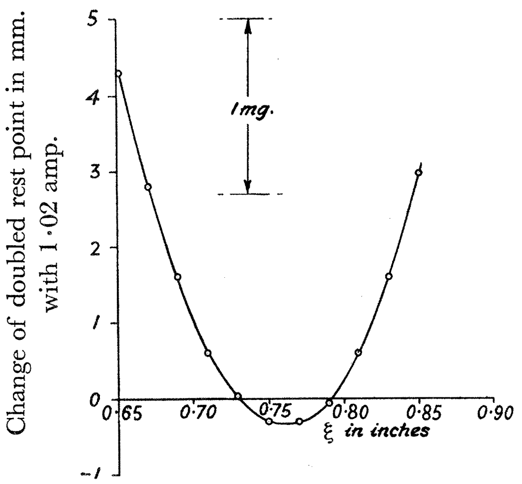

FIG. 8—Adjustment of the height ξ of the large right-hand cylinder for maximum force.

When each large coil is coaxial with its small coil, the axial adjustment is effected for each large cylinder in turn, in just the same way as before, except that the cylinder is displaced vertically instead of horizontally. One curve obtained for the right-hand system is reproduced in fig. 8. Its latus rectum is, in accordance with theory, half that of the curves obtained in the adjustment for coaxiality. In all, eight parabolae are obtained in the process of adjustment, and each can be used to determine $\partial^2 F/\partial z^2$ if the working current be known approximately. This was in all cases about 1·0186 ampere, and the mean value of $\partial^2 F/\partial z^2$ obtained in this way is $-$ 24·80 dynes/cm./cm. for 1 ampere, which, bearing in mind the limitations of the method, is in good agreement with the theoretical value $-$ 25·05 dynes/cm./cm.

The vertical adjustment just described gives ξ_M, the position in which the sum of the forces is a maximum. To find ξ_D, the position of the large cylinder for which the forces exerted by its upper and lower windings are equal, current is maintained in the same direction in all the windings of the system under adjustment, whilst the large coil of the other system is made inoperative by opposing its adjacent helices. It is not convenient to oppose the helices of the second small coil, which is therefore acted on by the large coils of the first system; but after one set of observations the current in all the coils of the system under study is reversed; the secondary effect is thereby reversed, whereas the primary force remains unchanged. The first large cylinder is displaced axially by small amounts, and the corresponding rest points are observed.

This gives the two lines of fig. 9.

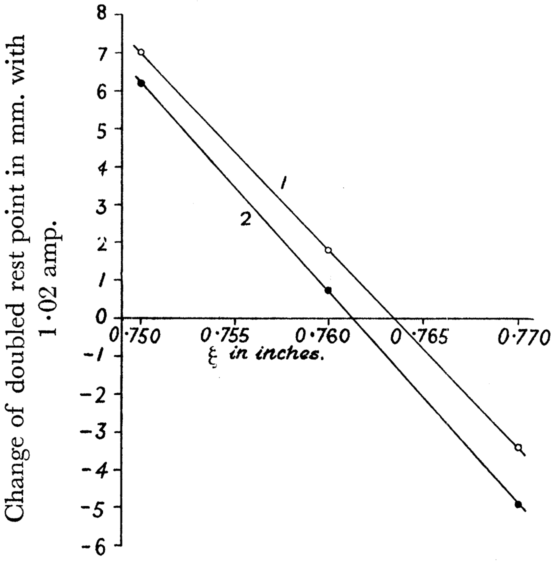

Fig. 9—Adjustment of the height of ξ of large right-hand cylinder for equality between forces exerted by its upper and lower coils.

From the graph we obtain the slopes s_1 and s_2 of the two lines, and the abscissae ξ_1 and ξ_2 of their intersections with the axis of ξ, then ξ_D, $\partial F/\partial z$ and $\partial G/\partial z$ can be deduced as follows:*

As before denote by ξ the height of some reference plane of the large cylinder under adjustment, measured from an arbitrary zero, by ξ_D and ξ_S the values of ξ for which F_u is equal to F_l, and G_u equal to G_l respectively, and by μ any small

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* This question and several others are dealt with in greater detail in a paper shortly to be published in the 'Collected Researches of the National Physical Laboratory.'

out-of-balance weight which, when there is no current in the coils, causes the beam of the balance to be slightly out of the horizontal. Let 2L be the length of the beam between the outer knife edges, let M be the mass of the moving parts, *i.e.*, the beam, the suspended coils, and all their fittings, let h be the distance between the centre of gravity of the moving parts and the fulcrum, and call θ the angle which the beam makes with the horizontal in the position of equilibrium, when the current is maintained in the coils according to the scheme described above. Then

$$\mathrm{M}gh\theta - \mu g\mathrm{L} - 2\mathrm{L}\,(\xi - \xi_{\mathrm{D}} - \mathrm{L}\theta)\,\frac{\partial \mathrm{F}}{\partial z} \pm 2\mathrm{L}\,(\xi - \xi_{\mathrm{S}} + \mathrm{L}\theta)\,\frac{\partial \mathrm{G}}{\partial z} = 0\,. \qquad (24)$$

In the equation the sign of the last term of the left-hand member depends on the relative directions of the currents in the system under adjustment, and in the suspended coils of the other system.

If we measure θ for different values of ξ and plot the results on a graph, we obtain, as 24 shows, a line of slope

$$s = \frac{\delta\theta}{\delta\xi} = \frac{\dfrac{\partial \mathrm{F}}{\partial z} \mp \dfrac{\partial \mathrm{G}}{\partial z}}{\dfrac{\mathrm{M}gh}{2\mathrm{L}} + \mathrm{L}\left(\dfrac{\partial \mathrm{F}}{\partial z} \pm \dfrac{\partial \mathrm{G}}{\partial z}\right)} \qquad (25)$$

To find Mh/2L we perform a sensitivity measurement, *i.e.*, we transfer a known mass μ' from one scale pan to the other, and observe the change in deflexion Δ. Then from 24

$$\mathrm{M}h\Delta = 2\mu'\mathrm{L}\,. \qquad (26)$$

Writing σ for Mh/2L, denoting by s_1 and s_2 the two values of s given by 25, *i.e.*, the slopes of the two lines obtained by experiment, we find, neglecting $(\partial\mathrm{G}/\partial z)^2$ in comparison with $(\partial\mathrm{F}/\partial z)^2$,

$$s_2 + s_1 = \frac{2\partial\mathrm{F}/\partial z}{g\sigma + \mathrm{L}\partial\mathrm{F}/\partial z}, \qquad (27)$$

and

$$s_2 - s_1 = \frac{2\,\dfrac{\partial \mathrm{G}}{\partial z}\left(g\sigma + 2\mathrm{L}\,\dfrac{\partial \mathrm{F}}{\partial z}\right)}{\left(g\sigma + \mathrm{L}\,\dfrac{\partial \mathrm{F}}{\partial z}\right)^2}, \qquad (28)$$

these equations yield $\partial\mathrm{F}/\partial z$ and $\partial\mathrm{G}/\partial z$. When the current is i instead of being unity, the differential coefficient in 27 and 28 must be multiplied by i^2.

Suppose, moreover, that by means of a rider or otherwise the equilibrium position of the beam is made to be horizontal when there is no current in the coils. This is equivalent to making μ zero in 24; then, by putting θ equal to zero in that equation, and solving for ξ, we obtain the values ξ_1 and ξ_2 at which the two lines of fig. 9 cut the axis of ξ. With the approximation made above as regards $\partial G/\partial z$, this gives

$$\xi_2 + \xi_1 = 2\xi_D, \quad \ldots \ldots \quad (29)$$

$$\xi_2 - \xi_1 = \frac{2\,(\xi_S - \xi_D)\ \partial G/\partial z}{\partial F/\partial z} \quad \ldots \ldots \quad (30)$$

In this way the value of ξ_D, which was required, is obtained in a simple manner from equation 29, whereas equation 30 gives the difference between the heights above the ground of the two suspended coils.

From fig. 10 we find 63·00 and 0·98 for $\partial F/\partial z$ and $\partial G/\partial z$, whereas the calculated values are 62·25 and 1·08 dynes/cm. for 1 ampere.

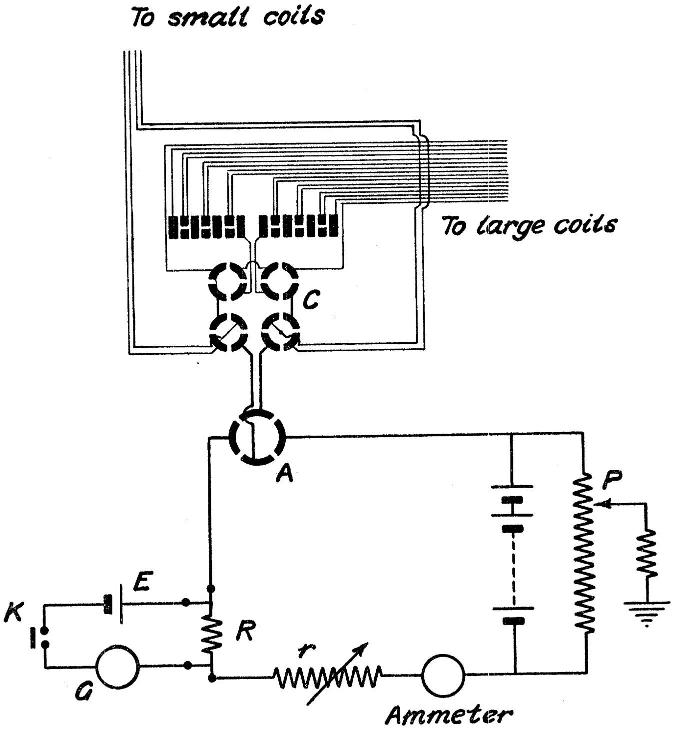

FIG. 10—Circuit for current balance.

Using the experimental values and formula 30, we find that $\xi_S - \xi_D$ is $-$ 1·8 mm., which means that the right-hand suspended coil is at a slightly higher level than the left-hand suspended coil. As the difference is small, and does not in any case introduce errors, a further adjustment would have been unnecessary.

THE ELECTRIC CIRCUIT

(a) *Circuit Arrangements*—The circuit is shown diagrammatically in fig. 10. All the go and return leads between the commutator switch A and the balance coils are twisted together and there is interposed a plug and commutator board

C, which controls the direction of the current in the various coils. The lead from A to the standard 1 ohm resistance R is carefully insulated, so as to ensure that the current through the balance coils is the same as that through R. The variable resistance *r* serves to keep the current to a value such that the difference of potential between the terminals of R just balance the E.M.F. of the standard cell E. Fine adjustment is effected by moving a slide which short-circuits part of a resistance formed by mercury in a narrow groove cut in keramot. With a battery of 500 ampere-hours' capacity it is easy to keep the maximum current variations under 1 part in a million throughout a determination lasting about half an hour. The average variation is, of course, much less.

All parts of the potential circuit are highly insulated. The potential circuit is not connected to earth, because in order to compensate certain electrostatic attractions it is desirable to earth the battery at some other point. The pedestal of the balance and all the metal portions of the fixed parts of the balance are connected to earth.

The insulation of the various parts of the circuit is tested at 30 volts, before and after a series of determinations of current. The insulation of some parts of the circuit, especially that between adjacent helices of the marble cylinders, is greatly influenced by atmospheric conditions. It is at its best in winter, when the room is artificially warmed and the air thereby kept dry, the value being then greater than 10,000 megohms. Even in summer, however, it never falls to a value low enough to throw any doubt on the accuracy of the measurement of current. The insulation resistance between the twin leads of each coil is measured separately; its value is of the order of 20,000 megohms. The insulation between the standard 1-ohm coil and earth is too good to admit of measurement with the galvanometer used. All the balance coils together have a resistance of 10,000 megohms to earth, and the resistance to earth of the leads of the standard cell is twice as large. The resistance between the two leads is also about 20,000 megohms. It is important to keep this resistance high, because the resistance of the standard cell is appreciable, lying between 500 and 1000 ohms. With an insulation resistance of only 1000 megohms between the leads, the potential difference at the terminal would be between $0 \cdot 5$ μV and 1 μV lower than the E.M.F. of the cell. The insulation between the galvanometer and the earth is tested by connecting the galvanometer terminals, in turn, to a terminal of the resistance R, fig. 10; at no time has a deflexion been observed.

Since thermo-electromotive forces in the potential circuit can be kept below 1μV, which is smaller than the order of repetition of successive determinations of current, little would be gained by arranging for reversal of the leads to the standard cell and to the resistance half-way through a determination. Instead, efforts have been made to reduce parasitic E.M.F.'s to the lowest possible value. With this object in view, copper has been used for the connexions and terminals, and, as far as possible, for the instruments, in the potential circuit.

(b) *Standard Cells.*—The cells used, all made at the National Physical Laboratory at different times, are cadmium Weston cells, the cadmium sulphate being dissolved in decinormal sulphuric acid. They are often compared with the groups of cells by which the unit of E.M.F. is preserved at the N.P.L., and have never been found to change by more than 1 or 2 μV. The E.M.F. of decinormal acid cadmium cells is known to be 62 μV lower than that of neutral cells made from the same materials. This correction and the temperature correction, which is well known, have been applied whenever it has been considered desirable to refer the results to the "neutral Weston cell at 20° C."

(c) *Standard Resistors*.—Three different resistors were utilized in the course of the investigations. The first was an open coil of manganin strip wound in 1904. It exhibited irregular changes in resistance, sometimes amounting to a few parts in a million in the course of a week, and was in consequence discarded. The sealed coil, No. L 641, of manganin wire, was then used, but was not very satisfactory for the relatively high working current of 1 ampere. It was therefore replaced by a coil of manganin strip, L 710, wound especially to fulfil the conditions required.

PARASITIC EFFECTS

It was verified experimentally that electrostatic attractions between the various portions of the coils do not affect the final result by even 1 part in 10 million.

There is also an attraction between each suspended coil and a copper disk fixed a few millimetres below it; the disk is a safety device, designed to stop the coil falling right down, in the event of its becoming accidentally unhooked from its supports. The attractions of the two coils at each end of the beam can, however, be made to compensate each other by earthing the battery at a point such as to equalize the mean potentials of the suspended coils relatively to earth. The correct earthing point was determined experimentally. Whether compensated or not, however, the effect never causes an error in the measurement of current, because readings are always taken with the current in the large coils reversed, and this reverses the electromagnetic, but not the electrostatic couple.

The procedure of reversing the current in the large coils also eliminates the electromagnetic forces exerted on the suspended part of the circuit by all leads fixed in position, except from the large coils to their individual commutators. It was, however, verified that even these leads do not influence the result. They were disconnected at the terminals on the large cylinders, and the twin leads of each pair were connected together by short links. The balance was used in the normal way with current in the small coils, and the change of rest point on reversing the current in the short-circuited leads was observed. The change was so small that it could hardly be measured.

The effect of the fine flexible silver connectors joining the fixed to the moving parts of the circuit, some of which can be seen at S in fig. 3, was investigated in an analogous manner, and was also found to be hardly measurable. The measurement of current is not affected by more than 1 part in 2 million by those connectors.

Magnetic effects were guarded against by employing non-magnetic material throughout for the construction of the balance. The room in which it is erected is also free from iron and steel. All the materials used for the balance and its pedestal (marble, wire, etc.) were tested. The bulky parts, and those likely to have more influence owing to their proximity to the coils, as, for instance, the marble, have a permeability differing from that of air by not more than 1 part in 100,000. For the other parts, the difference is in most cases less than 2 in 100,000, but occasionally higher values were allowed for small fittings remote from the strong regions of the field.

Moreover, an extensive investigation of the effect of neighbouring masses of iron was conducted. It revealed that the measurement of current was not affected by more than 1 part in 4 million by a block of steel weighing 12 kg., placed below one coil system at a distance of about 120 cm. from the mean diametral plane.

The possibility of opposing one coil system to the other, and of making one or more coils inoperative by opposing adjacent helices, greatly facilitates the study of parasitic effects and the adjustment of the coils into their correct position. All these operations can be effected without keeping the current rigidly constant, for the difference between two nearly equal forces is not appreciably altered by a small change of each force in the same ratio.

MEASUREMENT OF CURRENT

In order to maintain at a constant value the current measured by the balance, it is passed through a resistance R of known value, and is adjusted so that the p.d. at the terminals of the resistance is equal to the E.M.F. E of a standard cell. The current is then equal to E/R, and when R is known E can be found.

After the various insulation tests have been performed, a current of roughly 1·02 ampere is maintained in the standard resistance and the adjustable resistance for about an hour, in order to bring them to a steady temperature before starting the measurements. The barometer is read, the dew point of the air in the balance case is measured, and the balance beam is lowered, but prevented from swinging by the agate stops. The oil in the baths of the standard cell and of the standard resistor is stirred, and the thermostatic control of the bath containing the resistor is set in action. The readings of the spirit levels and of the screw heads controlling the positions of the large coils are taken, to make sure that the coils have not shifted from the positions in which they were set.

After about an hour the sensitivity of the balance is measured, the rest point* is adjusted by means of a rider or otherwise to 100 mm. (which corresponds to the horizontal position of the beam), the thermometers in the oil bath, in the balance

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[*] When determining the rest point by observing oscillations, it is convenient to add to one extreme reading the mean of the extreme readings on the other side immediately preceding and following it. The figure obtained in this way is twice as large as the scale reading corresponding to the actual rest point, and will, to avoid confusion, be called the "doubled rest point."

case, and in the marble cylinders are read, the current is switched on to the balance coils and kept constant by alteration of resistance, the appropriate weight is added to one scale pan, the beam is freed, and the rest point is observed. After this the beam is steadied by the agate stops, but not lifted, the galvanometer key is released, the commutators of the fixed coils are operated, the galvanometer key is depressed and the current is adjusted anew, the first weight is lifted off its pan, and another placed on the other pan, and the rest point is again observed. This procedure of reversal is repeated six or seven times, after which the two commutators of one of the systems of coils are operated, and another six or seven observations of rest point are made as before, the current in the fixed coils being reversed between successive observations and two new weights being used. The two sets give the sum and the difference of the direct and secondary forces, so that the latter can be eliminated by addition. The whole experiment occupies about half an hour. On its completion the temperatures and the dew point in the balance case are again measured, and the barometer is read.

After the various corrections for temperature, atmospheric pressure, etc., have been applied, the current is obtained from the instrument constant given on p. 142. As, however, it is more convenient for purposes of comparison to express the result in terms of the E.M.F. of a standard cell, the current is multiplied by the resistance, and the product corrected for the temperature and acidity of the particular cell employed, so as to give the E.M.F. of the " neutral Weston cell at 20° C."

RESULTS

Over 120 independent determinations have been made during the last five years. They are plotted in fig. 11 and are grouped according to the time at which they were made and the resistor which was employed. The figures in circles indicate the number of observations in each group, the other figures the mean deviation in parts per million. It is usually possible to make two determinations in a day, and each group represents a relatively short interval of time, from one to four weeks. The last group, of 16 observations, is an exception. In this case the observations were spread over more than four months, and the conditions were purposely changed in several respects. For example, in a few determinations one coil system only was used, in order that the conditions might approximate to those obtaining in the current balance of the National Bureau of Standards, in which the system of coils is not duplicated. The fact that the mean deviation from their mean of the results of this group is only two parts in a million shows that the current balance repeats to a high order of accuracy. It will be recalled that the electromagnetic force is only 4 gm.-wt., whereas the beam, suspended coils, and fittings weigh over 14 kg.

Repetition over an interval of a few years has not proved so satisfactory, and deserves some discussion. All the results plotted in fig. 11 are referred to the instrument constant calculated from the linear dimensions measured at the end of

1932. The dimensional changes which occurred during the period 1928–32, although relatively large, happened to be roughly proportional to the dimensions concerned, and the resulting change in the instrument constant was negligible. Still, it follows neither that even approximate proportionality held throughout the interval, nor that it holds good for the changes which have taken place since the end of 1932. In fact, the 1935 check measurements of linear dimensions show that the last point on the curve of fig. 11 should be about 4 μV lower than it is, which would make the curve steeper still.

It is nevertheless realized that a change in the instrument constant is not essential to an explanation of the distribution of the points of fig. 11. The first three points may not be so accurate as the others, not only because greater experience was acquired in the handling of the instrument as the work proceeded, but also because, for the determinations corresponding to the first three points, the large coil was not coaxial with the small one, but was set at a point corresponding to the axis of one of the parabolas of fig. 7, instead of at the reading midway between the axes of the two curves. This necessitated the application of a correction of − 19 parts in a million to the values of E.M.F.

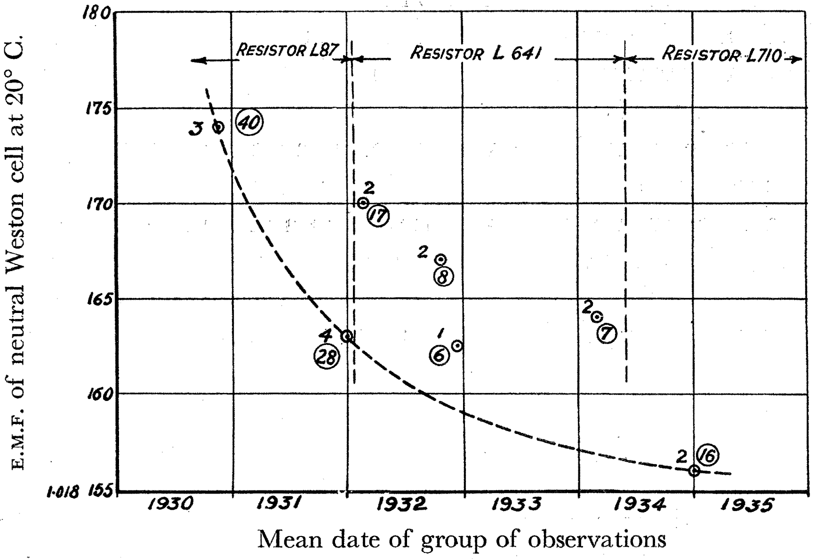

Fig. 11—Summary of the results of the measurement of the E.M.F. of the neutral Weston cell, showing date and resistor used for each group. The figures in circles indicate the number of observations in each group, the other figures the mean deviation in parts per million.

The factor responsible for the major part of the discrepancies, however, seems to be the change of resistor. For reasons already given, it is believed that L 641 is not so reliable as the other two resistors.

Taking all those factors into consideration, and pending more measurements with the new resistor, we take 1·018 16 as the most likely value.

The accuracy of the final result is affected mainly by three different causes, viz., systematic errors in weighing and in the electrical operations, want of exact knowledge of the linear dimensions of the coils, and uncertainty in the value of the acceleration due to gravity.

Had it not been for the gradual diminution in the values obtained from 1930 to 1935, and the possible influence of the resistors on the final result, which fig. 11 might be considered to suggest, the tendency would have been to assess the systematic errors at a few parts only in a million. As things stand, however, and pending the completion of more measurements, it is felt that 1 part in 100,000 must be allowed for this cause of error.

The second cause or error arises from want of exact knowledge of the linear dimensions of the coils. The gauges used for these measurements can be referred to one another with an accuracy superior to $0\cdot1\mu$, and, in the current balance problem, the unit of length to which they are referred does not influence the result. Moreover, the experimental work on the difference between the forces exerted by the two systems of coils shows that the irregularities of the windings have been adequately allowed for in the calculations.

There remains the uncertainty in the dimensions of the coils when in actual use. The several determinations of current were made at room temperatures ranging from 11° C. to 22° C.; as the final results show no dependence on temperature, it is deduced that the effect of the temperature changes of the marble cylinders are adequately allowed for by applying the correction determined from the thermal coefficients of linear expansion. A given error in the determination of the actual temperatures of the cylinders during the measurements would, it is true, give rise to a constant error, but a consideration of the methods employed for determining the temperatures shows that the error due to this cause cannot amount to more than 3 or 4 parts in a million in the final result. It is therefore estimated that an allowance of 1 part in 100,000 in the final result is adequate for the uncertainty in the instrument constant.

The third cause of error is the uncertainty in the value of the acceleration due to gravity. This has been taken as $981\cdot195$ cm./sec./sec. at Teddington. This figure is based on the Potsdam absolute value, which is at present almost universally used as basis of reference. Although the difference between Potsdam and Teddington is known to a few parts in a million, the absolute value itself is not known to that accuracy. An error of 2 parts in 100,000 is therefore allowed on the above figure, giving an uncertainty of 1 part in 100,000 in E.M.F. or current.

At present, therefore, the balance may be considered capable of an absolute accuracy of 3 parts in 100,000. But when comparing results obtained on this balance with results obtained in other laboratories which also use the Potsdam value as basis for the acceleration due to gravity, the accuracy can be assessed at 2 parts in 100,000.

Since 1925 the unit of resistance in use at the National Physical Laboratory has been maintained by a number of coils of wire. The mean value of these coils was determined in terms of the international unit by means of mercury tubes in 1912 and in 1925. The two values found differed by 25 parts in a million, and their mean has been used for the purpose of the present investigation.

The present investigation therefore shows that, in terms of the ampere and of the international unit of resistance, as preserved at the National Physical Laboratory, the E.M.F. at 20° C. of the neutral Weston cell, indirectly realized at the Laboratory by means of acid cadmium cells, is 1·018 16 ± 0·000 03. The determinations made with silver voltameters in Berlin in 1931 having yielded 1·018 30 ± 0·000 02 for the same quantity expressed in terms of the international ampere, we arrive at the figure 1·000 14 ± 0·000 05 for the ratio of the ampere to the international ampere.

USE OF THE BALANCE AS A "CURRENT PENDULUM"

As early as 1911 F. E. SMITH had proposed to use the balance as a pendulum, and experiments had been made in which the current was maintained in the coils in directions such as to cause no appreciable change in the equilibrium position, but a large change in the restoring couple. From the alteration in periodic time, the current can be calculated, provided the moment of inertia of the moving parts is known.*

The balance is now provided with a device for determining this quantity, but serious difficulties are introduced by the large temperature coefficient of the periodic time, and an accuracy of 1 part in 10,000 is as good as can be obtained at present. The possibilities of the method are being further investigated.

The investigation was carried out at the National Physical Laboratory, and thanks are due to the Superintendents and Staffs of the Departments of Engineering, Metrology, and Electricity for their cooperation ; to Sir JOSEPH PETAVEL, Director of the Laboratory, for the encouragement received from him and the sustained interest he took in the work ; and to Sir RICHARD GLAZEBROOK, Sir FRANK SMITH, and other members of the Electrical Units and Standards Committee of the Laboratory for criticism and advice.

SUMMARY

The current balance erected at the National Physical Laboratory in 1905 by AYRTON, MATHER, and SMITH was overhauled, new coils were constructed, and numerous measurements of current were effected between 1930 and 1935. The results indicate that the ampere is 14 ± 3 parts in 100,000 greater than the unit of current realized at the Laboratory by means of resistance coils and cadmium cells. This value for the ampere exceeds by 2 parts in 100,000 the value found by AYRTON, MATHER, and SMITH in 1907.

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* 'N.P.L. Ann. Rep.,' 1920, p. 58 (H.M. Stationery Office).

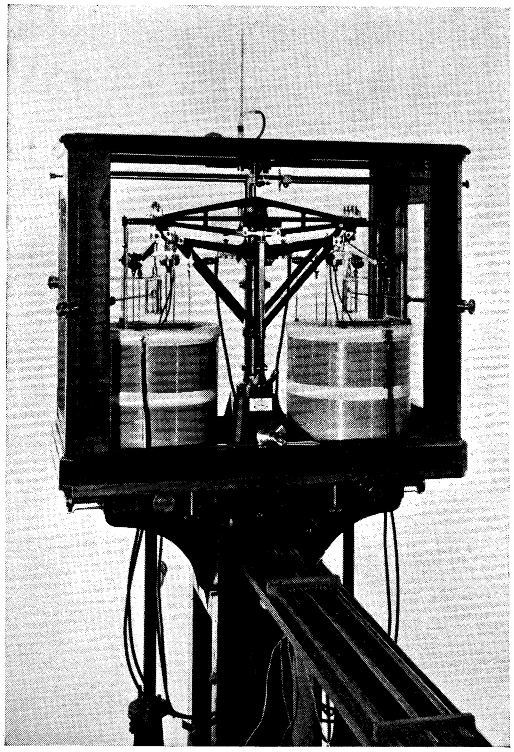

Fig. 1—Current balance of the National Physical Laboratory.

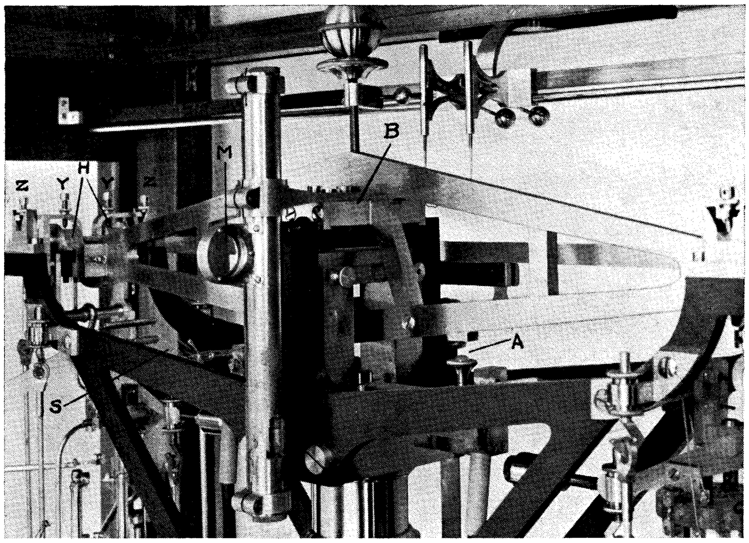

Fig. 3—Photograph showing tubular fitting added to the beam, and other details of the balance.

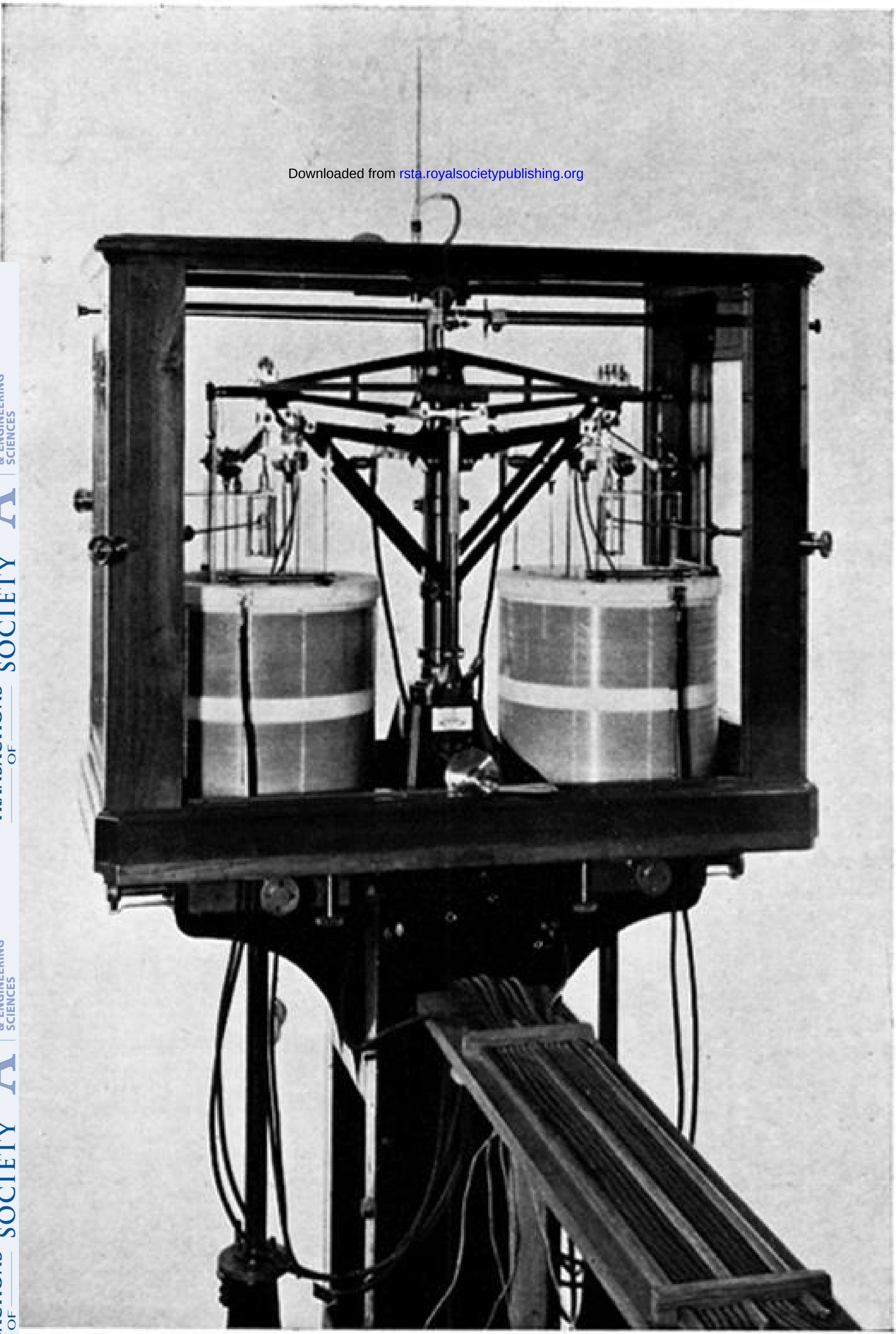

FIG. 1—Current balance of the National Physical Laboratory.

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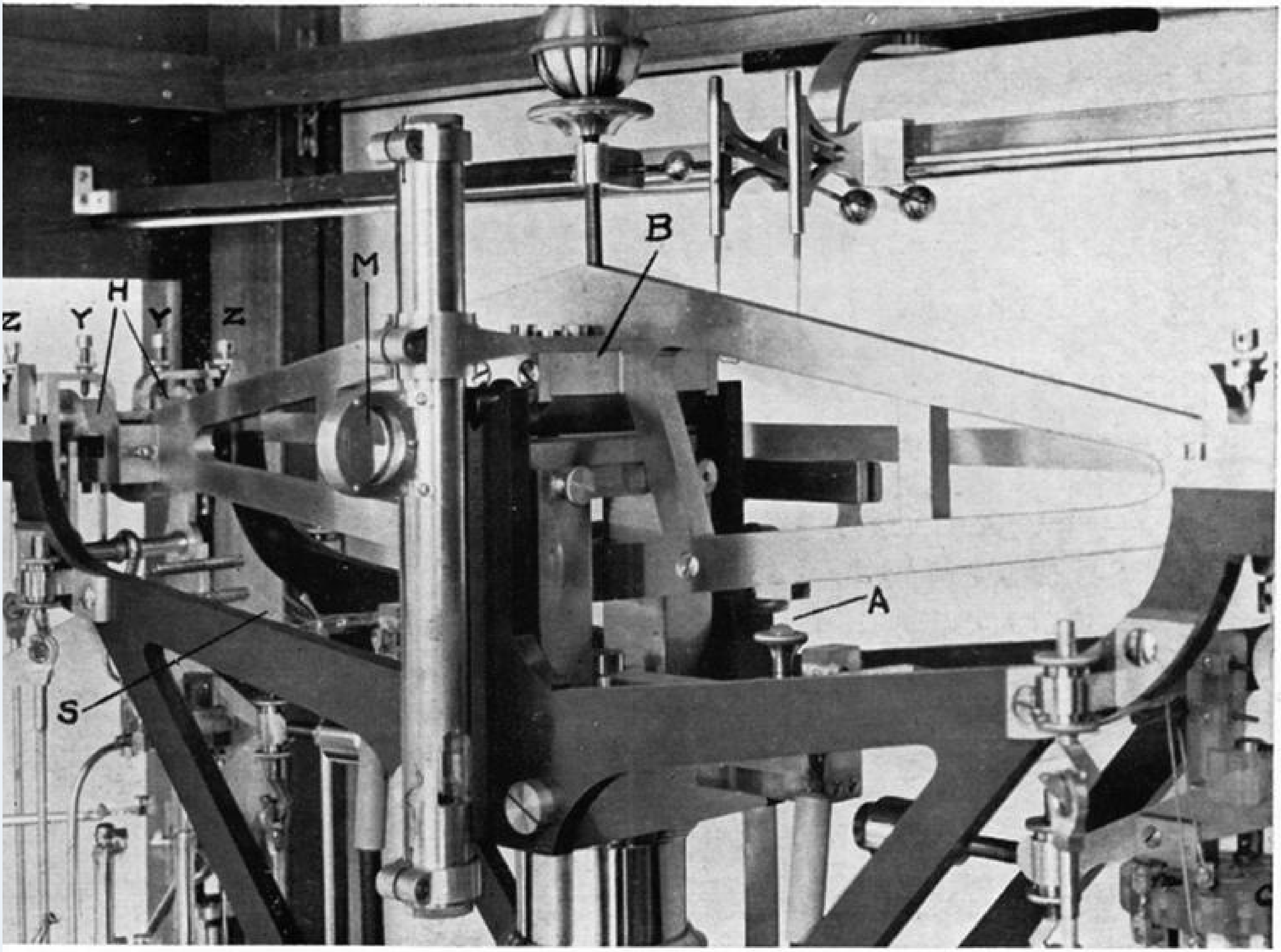

IG. 3—Photograph showing tubular fitting added to the beam, and other details of the balance.